LINKS ANALYSIS IN SOCIAL NETWORK USING STATISTICAL MECHANICS

Author:Klotilda NIKAJ

klotildanikaj@hotmail.com

Department of Physics,University of Tirana, ALBANIA

11th Young Researcher Meeting

IPN, Trento, Italy, September 06-09, 2021

INTRODUCTION

In network theory, a new type of mathematical models of human society, has been inspired by analogies between social behavior and statistical mechanics. Link analysis is a data analysis technique used to evaluate relationships between nodes. To find the most influential node or the list of nodes in the networks[1,3,4], link analysis were devised to solve the problem in the past. To determine whether two network embeddings are similar, we start from the linking number that measures the number of times two closed cycles wind around each other, capturing the number of tangles[2,3,4]. The graph linking number, which for a network with embedding ε represents the sum of the linking numbers of all pairs of cycles in the graph. In this paper, specific application of the formalism is explored in the context of the social networks optimization. Social networks derived from real world phenomena are sparse, so inferring random edges is expected to have low accuracy. When networks evolve, the number of possible links grows quadratically while it is expected that new edges grow in a linear fashion with added new nodes. Several analogues with the statistical mechanics treatment of thermodynamics have been made.

MATERIALS AND METHODS

A dynamic social complex system

•Expensive computationally

•Software implementation

•Requires complete dataset

•Requires static dataset

THE PROPOSED MODEL

➢Define a network with N agents.
At each time step the graph linking number, which for a network with embedding represents the sum of the linking numbers of all pairs of cycles in the graph[3] :

$$G(\varepsilon) = \sum_{c,c' \in \{C\}} l(\varepsilon, c, c')$$

➢The partition function for this ensemble sums over all possible paths for all links with a fixed set of node locations, integrated over all possible node locations is:

$$Z = \sum_{\{Link\ paths\ \gamma\}} \sum_{\{node\ position\}} \exp[-\beta V_{el}] = \int \prod_{i=1}^{N} d^3X_i \prod_{l=1}^{N} d\gamma_l \exp < -\beta V_{el} >$$
$$= \int \prod_{i=1}^{N} d^3X_i\, Z_L[\{X_i\}]$$

➢The average of the energy of the ensemble can be expressed as:

$$< E >= -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = E_0 + \sum_n C_n < \varepsilon_n >,$$

➢The total elastic energy of a network layout with links is well approximated by:

$$E(G) \approx E_0 + G\varepsilon$$

➢The Boltzmann distribution for a network with elastic energy is:

$$P(E(G)) = \frac{n(G)e^{-\beta E(G)}}{Z}$$

➢And the expected entropy: $S = -N_p \log\left(1 - e^{-\beta\varepsilon}\right) + \frac{N_p \varepsilon\beta}{e^{\beta\varepsilon} - 1}$

REFERENCES

[1]Barrat, A., Barthelemy, M. & Vespignani, A. Dynamical Processes on Complex Networks (Cambridge Univ. Press, 2008).
[2]Bianconi, G. & Capocci, A. Number of loops of size h in growing scale-free networks. Phys. Rev. Lett. 90, 078701 (2003).
[3]Parisi, G. Te physical meaning of replica symmetry breaking. Preprint at https://arxiv.org/abs/cond-mat/0205387 (2002).
[4]Albert, R. & Barabási, A.-L. Statistical mechanics of complex networks. Rev. Mod. Phys. 74, 47–97 (2002).

RESULTS AND DISCUSSIONS

Lets consider the model by using complex topologies of the agent network, i.e, scale free networks generated by the Barabasi-Albert model[2] and small world networks by the Watss-Strogatz model[3], starting with N=10000 agents and with an average degree

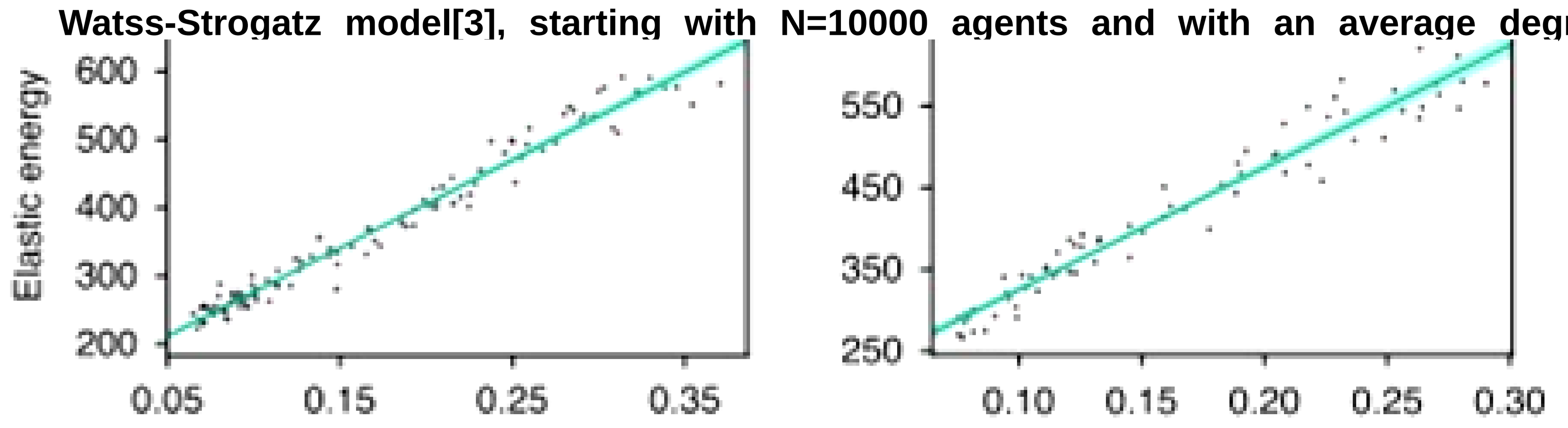

Figure 1. Represent the elastic energy versus G(n) for networks generated by Erdos Reny (left) and Barabasi Albert (right), for n=1000 and <k>=8

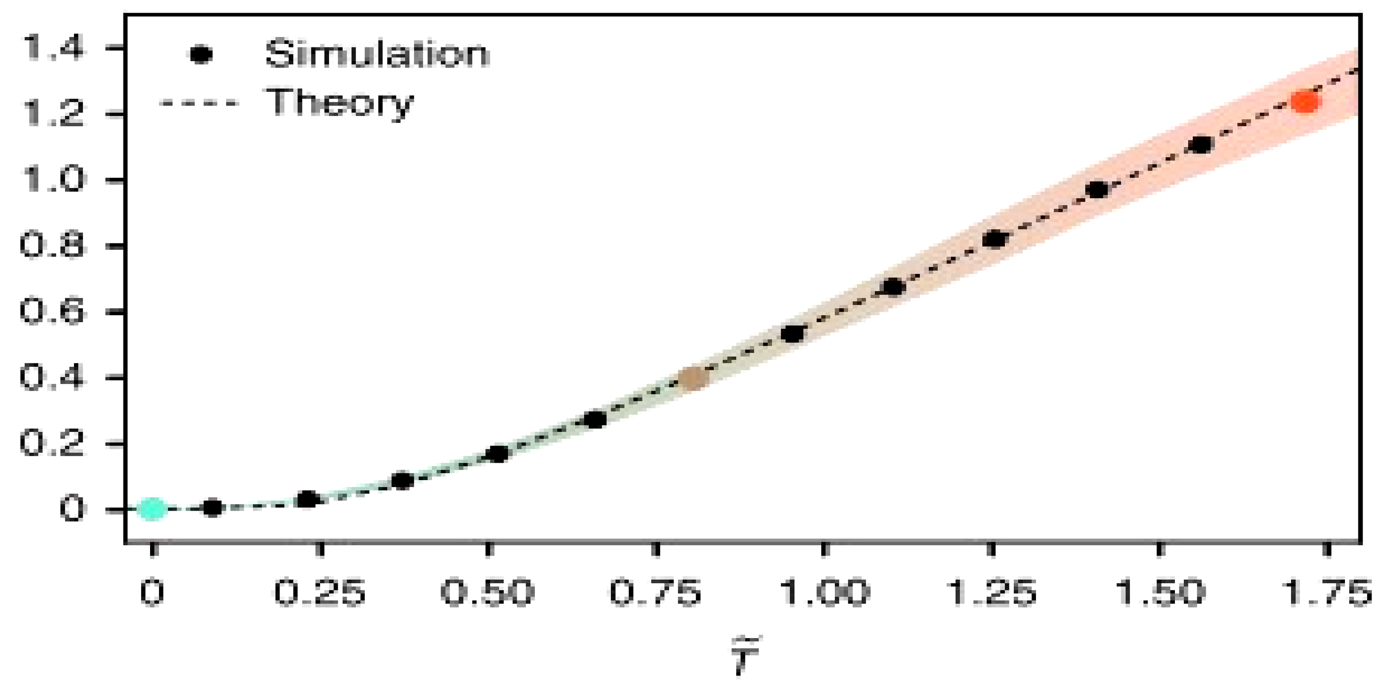

Figure 2. For **different temperatures, while keeping the node positions fixed, illustrating that higher T corresponds to more curving of the links. In turn, the more curved links lead to more tangles, resulting in a higher G(n) Comparison between the analytic and numerical results for Gn for lattice layouts at different temperatures (different canonical ensembles). The dots are the mean Gn(T)**

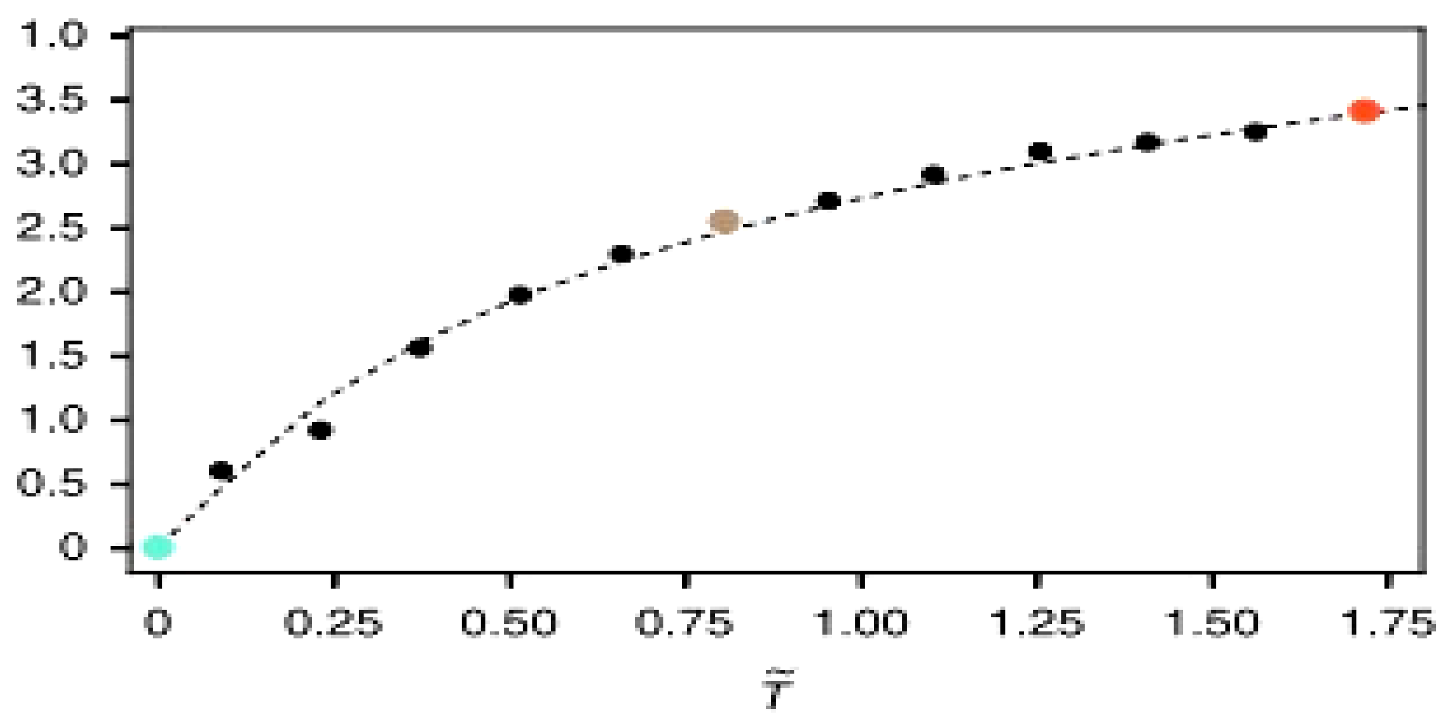

Figure 3. **Analytical and numerical results for the entropy for the canonical ensemble of the lattice layouts at different temperatures. The dots are the numerical results, and the dashed line is the analytical result.**

CONCLUSIONS

We find that a networks elastic energy depends linearly on the graph linking number, indicating that each local agent offers an independent contribution to the total energy.
In summary this statistical mechanical model allows us to systematically compare different network layouts, and determines the elastic energy of a physical network. Although our method measuring for a given network layout is computationally taxing, generally scales similar to a polynomial with the number of nodes, and at least quadratically with the number of links, which limits our ability to explore very large networks.

For any question please do not hesitate to contact me.
Thank you!